Wave Effects on Mega Ripples and Objects on a Sandy Seabed

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LONG-TERMS GOALS

Our general goals are to advance the hydrodynamics of waves, currents and the two-phase dynamics of sediment transport in coastal seas. An important part is to gain physical understanding of the physics and to enable quantitative prediction of the seabed evolution near the shore. The nearshore morphology includes features of a wide range of length scales: from the entire beach to sand bars and down to sand ripples. The mechanisms may differ over different scales, and may also be intertwined. This proposal focusses on the quantitative prediction and modeling of the formation of mega ripples on a sandy seabed under sea waves, with and without the presence of solid objects such as mines.

OBJECTIVES

During the first year of this two-year project, we attempted to extend past theoretical works on gently sloped ripples under purely oscillatory flows (i.e., waves of infinite length, Blondeaux &d Vittori, 1990, 1991) and under partially standing waves Mei & Yu (1997) to the evolution of mega-ripples under more complex waves, with and without a rigid object. In view of overwhelming evidence of steep ripples we have shifted emphasis on steep ripples. Specifically a mathematical model is nearly complete for

- (1) Flow over rigid ripples under: (1.a). waves of infinite wavelength, i.e., purely oscillatory flow, and (1.b) waves of finite wavelength.
- (2) Ripple formation on sandy bed: (2.a) Long-crested ripples emanating from a two dimensional cylinder under oscillatory flows. (2.b) Ripples formation and migration under periodic waves of finite length.

In this FY we have developed the numerical theory and completed the computational programs based on MATLAB. The computation time is so far quite demanding on a PC and too long for practical applications. We are nearing the end of writing the programs in C and will further make use of parallel computing clusters at MIT, in order that dozens of ripples in a wave length can be simulated.

APPROACH

We focus our attention to the coastal zone where the typical length scales are: water depth $5\sim10$ m; sea wave length 100 m; seawave amplitude 1 m; wave boundary layer thickness $1\sim10$ cm; sand ripple wavelength 10 cm ~1 m; sand ripple amplitude $1\sim10$ cm. While the slope of surface is assumed to

be small, the slope of ripples can be of order unity. The wavelength of waves is assumed to be much larger than the ripple length or height. Denoting the wavenumbers of and the ripples and the waves by k and K respectively, and the amplitude of ripples and waves by a and A respectively, we assume ka = O(1), $O(KA) = O(K/k) = O(\varepsilon) << 1$.

Fluid Flow. The flow about a few ripples lengths above the seabed is treated by the usual potential theory. For small amplitude waves the first and second order velocity field associated with the waves are easily found.

Directly above the sea bed, we use the fully nonlinear Navier-Stokes equations. In view of the anticipated slow evolution of sand ripples, the flow is assumed to satisfy the no slip boundary condition. To prepare for later extension to the three dimensional problem of a mine among ripples over the seabed, we work with the velocity components and pressure directly, instead of the stream function and vorticity. We then change to a curvilinear orthogonal coordinate system (ξ, η) in which the instantaneous rippled surface is imbedded as $\eta = 0$.

A splitting scheme of finite elements is used for time integration. The Fourier spectral method is use to analyze the tangential direction ξ , while staggered grids are used in the finite difference approximation in the vertical direction of η for the velocity components and the pressure field.

Sediment transport: Because the ripple slope can be steep, gravity force can be comparable to the bed shear stress. This situation is not covered by the usual empirical formulas of bedload transport obtained for essentially horizontal beds where the shear stress is far greater than the gravity force. We have modified the existing transport rate by first showing that the critical Shields parameter for a horizontal bed θ_{CR} should be replaced by the following:

$$\theta'_{Cr} = \theta_{Cr} \left[\cos \beta \left(1 + \frac{\tan \beta}{\tan \phi_S} \right) \right] \tag{1}$$

where β denotes the angle of inclination of the bed surface

$$\beta = \tan^{-1} \left(\frac{\partial h}{\partial x} \right) \tag{2}$$

and ϕ_S is the static angle of friction (~50 degrees). For bedload transport, the law of conservation of sand mass for bedload transport reads,

$$(1-n)\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{3}$$

where n denotes the porosity, h the ripple height about the mean seabed, and q the discharge rate. It can be shown that for steep sand surface q should be given by

$$\frac{q}{\sqrt{(s-1)gd^3}} = \frac{3.2S*}{\tan\phi_m} F(\beta)(|\theta| - \theta'_{cr})\{|\theta| - G(\beta)\theta'_{cr}\}\operatorname{sgn}\theta \tag{4}$$

where ϕ_m is the dynamical friction angle (~30 degrees) and

$$F(\beta) = \frac{\sin\phi_m}{\sin(\phi_m + \beta \operatorname{sgn}\theta)}, \qquad G(\beta) = \left(\frac{\tan\phi_m + \operatorname{sgn}\theta \tan\beta}{\tan\phi_s + \operatorname{sgn}\theta \tan\beta}\right)^{1/2}$$
 (5)

The bedload discharge formula Eq. (4) holds even when the local bed slope $\frac{\partial h}{\partial x}$ is up to the angle of repose. This new formula, obtained recently by Yile Li in this PhD study, is vital to ripple prediction.

COMPLETED RESULTS

Rigid Ripples under pure oscillatory flows:

As the first application of the numerical theory we have calculated the flow over periodic rigid ripples. For low enough Reynolds number ($R = A\omega\delta/\upsilon$) the flow is periodic in time, as shown in Figure 1 for R=20, the flow is periodic. A strong vortex is seen to formed on the downstream side of a ripple crest twice in each period. Two vortices interact with each other and are transported upward by convection and eventually vanish by viscous dissipation. There exists a critical Reynolds number above which the flow becomes chaotic. Aperiodicity can be seen in Figure 2 for R = 35.

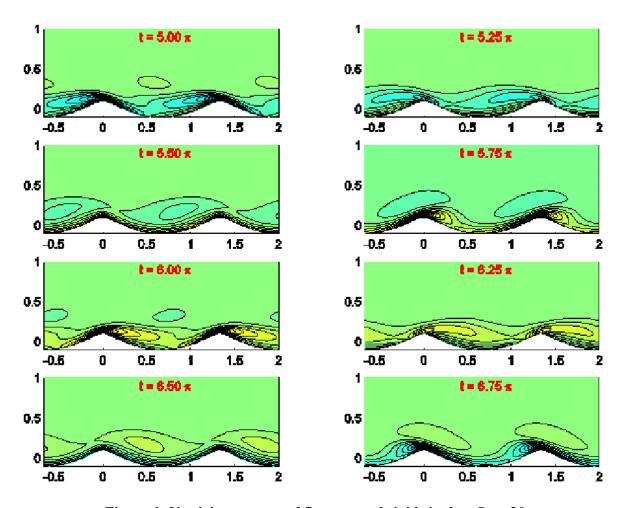


Figure 1. Vorticity contour of flow around rigid ripples, $R_{\delta} = 20$

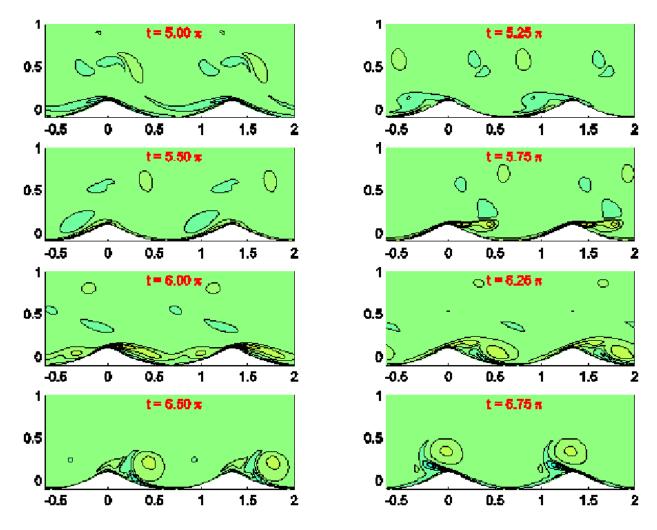


Figure 2. Vorticity contour of flow around rigid ripples, $R_8 = 35$

Ripple formation near a cylindrical obstacle.

Since the time scale for ripple development is much longer than the wave period, the ripple profile can be regarded as being rigid when the flow field is computed. The computed shear stress on the bed surface is used as the Shield parameter to compute the bed load and the new bed surface. After certain time the flow field is recomputed for the new bed geometry.

Figure 3 and 4 show the ripple evolution for an erodible seabed around a small bump (a half buried mine) at the center. The bed is initially flat everywhere else. In both cases the mobility number (related to the Shields parameter) is 7.36. Figure 3 is for a subcritical flow R=20, while figure 4 is for a supercritical flow R=35. In both cases the ripples are formed first around the obstacle and then spread away to the entire region with both wavelength and height increasing with the time until the equilibrium stage. For both R, the equilibrium ripples are essential periodi. The ripple-wavelength to surface-wave amplitude ratio is about $\lambda/A \sim 2$ and the ripple-height to wave-amplitude ratio is between $a/A \sim 0.15$ and 0.3. These values are quantitatively consistent with the experimental observations by Voropayev et al (1999), and the field data collected by Nielsen (1992).

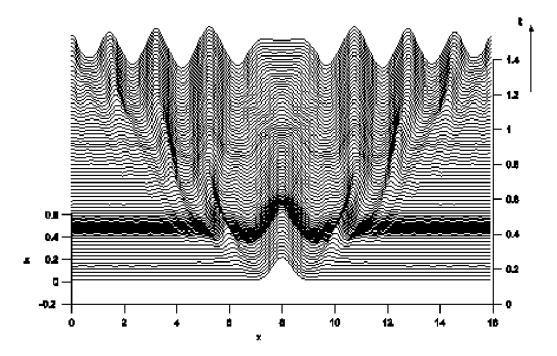


Figure 3. Ripple profile development, $R_{\delta} = 20, \psi = 7.36$

Ripple Profile Development, $R_{\rm p} = 50$, $\gamma = 7.85$

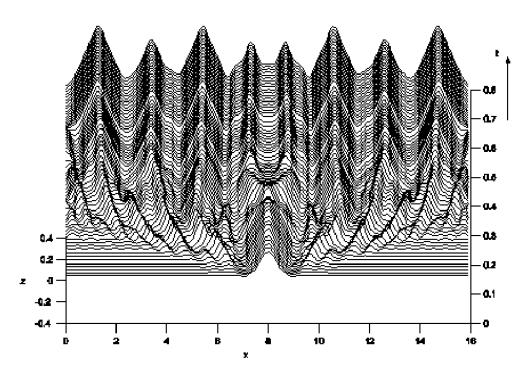


Figure 4. Ripple profile development, $R_{\delta} = 35, \psi = 7.36$

Rigid Ripples under Water Waves

Under surface waves of finite length sand ripples migrate in the direction of the dominant waves. Numerical computation must be carried out over at least one wavelength of surface waves. Pending further improvement of our algorithm, we have so far only calculated the fluid flow over rigid ripples. Figure 5 shows the development of vorticity close to the periodically rippled bed under a progressive surface wave advancing from left to right. The wavelength is only six times that of the ripples. Due to the phase delay of external flow over different ripples, the flow exhibits more complicated pattern than those under flow with infinite wavelength. The period-averaged mean shear stress distribution along the ripple profile is plotted in Figure 6. The peak values of mean shear stress occur in the close neighborhood of the ripple crests. But the positive peak on the left side of ripple crest is larger than the negative peak on the right, which implies a net force in the wave direction. When the bedload is accounted for (soon), this net force will surely lead to the migration of ripples under progressive waves.

IMPACT

Prediction of the fate of mines on a sandy seabed is not possible without a reliable computational model for the evolution of ripples under waves. Since the ripples in nature and in the laboratory are mostly steep and the flow nearby involve vortex shedding, past attempts on gentle rolling-grain ripples are of academic interest only. Also existing theory on the nonlinear development of gentle ripples rely on **weakly nonlinear** approximation valid where the flow is very near the threshold of instability. This is again of little practical significance since most of the interesting flows are far above this threshold. Therefore for both the hydrodynamics and the sediment transport, a fully nonlinear theory is a prerequisite to mine burial prediction. The future impact of the nonlinear models should therefore be certain.

WORK PLAN FOR THE FUTURE

Improving the speed of computation

The numerical code was first developed by using Matlab. It takes about 150 hours to reach the equilibrium state for a domain filled with 8-10 ripples. In a typical laboratory or field environment, there are 20-30 ripples within one water wavelength. The time for one run of ripple evolution may take 400-600 hours on a Pentium IV 2.4GHz PC. By improving the numerical scheme and rewriting the whole code in C, we are now reducing the total running time to ½ to 1/6, which still need 4-6 days to complete the simulation of one case. We are planning to take advantage of the existing parallel computing clusters to accelerate our numerical simulation. Later use of supercomputers is being considered.

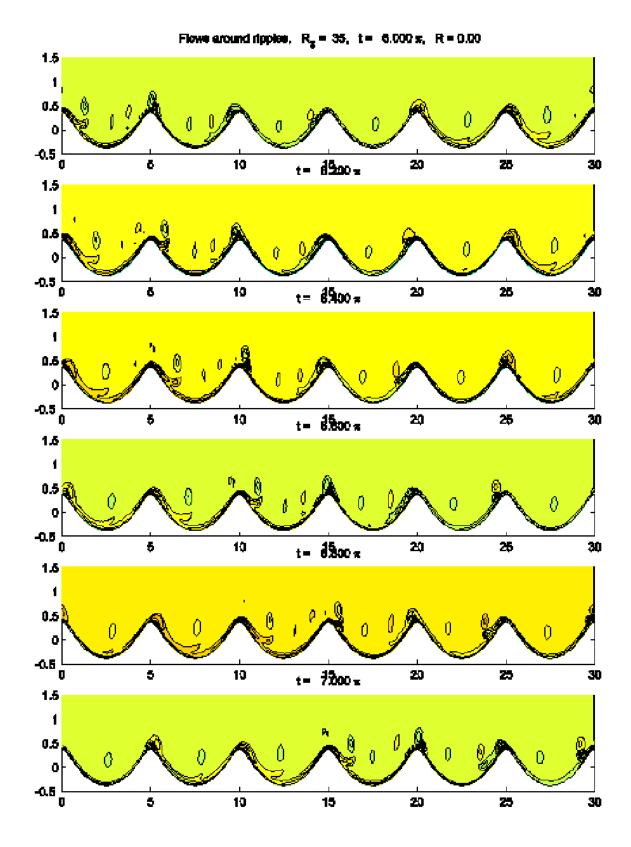


Figure 5. Vorticity contour of flow around rigid ripples under progressive surface waves, $\lambda_{wave}/\lambda_{ripple}=6, R_{\delta}=35$

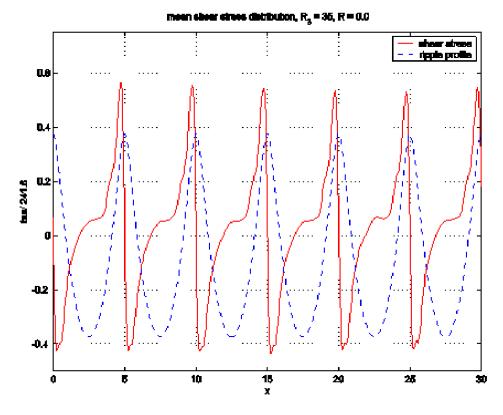


Figure 6. Steady shear stress distribution along ripples under surface waves, solid line – dimensionless mean shear stress; dash line – ripple profile, wave direction left to right, $\lambda_{wave}/\lambda_{ripple} = 6, \, R_\delta = 35$

Simulation of ripple evolutions under partially reflected waves

As has been shown in past works wave characteristics (wave length, direction, pattern) affects the formation of a sandy bed in crucial ways. We shall first calculate the formation of two dimensional ripples, both with and without the presence of a solid cylinder. The variation of the ripple height and length and the process of burial/unburial will be studied using the present mathematical model for steep ripples by assuming that the bed load dominates. Three tasks are planned

- (1) Pure oscillatory flows (infinite wavelength)
 - 1.a. Ripple initiation by instability and subsequent nonlinear growth towards final equilibrium.
 - 1.b. Ripple formation near a solid cylinder lying on initially plane bed surface. Initiation of ripples by vortices and final approach to equilibrium will be examined for various wave chagracteristics and cylinder spacing and sizes.
- (2) Progressive waves of finite wave length. In addition to the initiation of ripples, their growth and migration will be studied. The speed of migration will give us a good idea on how fast a mine will be buried or unburied.

(3) Partially standing waves. Here bars of wavelength about one half of the surface wavelength will be expected. The variation of ripple size and length between nodes and antinodes of the standing waves will be of great importance.

A more fundamental study is the effect of suspended load for which a new theory of fluidization is being initiated.

Laboratory Modelling

Blake Landry, a MS student supported in part by ASSERT, has joined the project by performing laboratory experiments on the formation of sand ripples and sand bars in a wave flume. The available wave flume at MIT has the length =80 ft, width =2 ft and depth = 3 ft. Preliminary tests with waves of length 6 ft. and reflection coefficient =0.2 have shown significant presence of sand ripples superimposed on bars. We are aquiring an acoustic array devise for measuring the sandy surface across the entire width of the tank. And will perform a series of experiments to check the theory.

RELATED PROJECTS

ONR N00014 90-J-3128 Dr.T. Swean, Wave-induced transport of sediments. There the emphasis is on the larger features of sand bars as well as other phenomena in littoral waters.

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PUBLICATIONS

0; 1 is in preparation.

PATENTS

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